## TOPIC: MECHANICS AND PROPERTIES OF MATTER

SUB-TOPIC: MACHINES
SPECIFIC OBJECTIVES

- Identify the three classes of levers.
- Define M.A, V.R and efficiency as applied to machines.
- Determine M.A V.R and efficiency of different types of machines.
- Describe factors that affect the efficiency of machines.
- Describe how efficiency can be improved
- Describe the applications of simple machines


## MACHINES

A machine is a device by means of which a force is applied at one point to overcome a force at some other point in order to ease work.

A force called effort is applied on a machine to overcome a resisting force called load.
A machine is used to;
(i) covert energy from one form to another.
(ii) amplify a force.
(iii) magnify movements.

## Terms used

1. Work in put

This is a work done by the effort and it is the product of the effort and distance moved by the effort.

Work in put $=$ effort $\times$ distance moved by effort
$W_{\text {input }}=\mathrm{E} \times \mathrm{D}_{\mathrm{E}}$

## 2. Work out put

This is the work done by the machine to overcome the load. It is the product of the load and distance moved by the load.

Work output $=$ load $\times$ distance moved by load.
$\mathrm{W}_{\text {output }}=\mathrm{L} \times \mathrm{D}_{\mathrm{L}}$

## 3. Mechanical advantage (M.A)

This is the ratio of load to effort.
Mechanical Advanyage $=\frac{\text { Load }}{\text { Effort }}$
M. $\mathbf{A}=\frac{\mathbf{L}}{\mathbf{E}}$

It has no unit because it is a ratio of forces (quantity with the same units)

## 4. Velocity ratio (V.R)

This is the ratio of distance moved by the effort to the distance moved by the load in the same time interval.
N.B velocity ratio $=\frac{\text { velcity of the effort }}{\text { velocity of the load }}$

It has no unit.

$$
\begin{aligned}
\text { Velocity Ratio }= & \frac{\text { distance moved by effort }}{\text { distance moved by load }} \\
& \mathbf{V} \cdot \mathbf{R}=\frac{\mathbf{D}_{\mathbf{E}}}{\mathbf{D}_{\mathbf{L}}}
\end{aligned}
$$

## 5. Efficiency

This is the ratio of work output to the work input expressed as a percentage.

$$
\eta=\frac{\text { work output }}{\text { work output }} \times 100 \%
$$

The ratio of mechanical advantage to velocity ratio also gives efficiency BUT it is not used a definition for efficiency.
$\boldsymbol{\eta}=\frac{\mathbf{M} . \mathbf{A}}{\text { V.R }} \times \mathbf{1 0 0} \%$
PROOF FOR
$\boldsymbol{\eta}=\frac{\mathrm{M} . \mathrm{A}}{\mathrm{V} . \mathrm{R}} \times \mathbf{1 0 0} \%$
From $\quad \boldsymbol{\eta}=\frac{\text { work output }}{\text { work output }} \times 100 \%$
$=\frac{\text { load } \times \text { distance moved by load }}{\text { effort } \times \text { distance moved by effort }} \times 100 \%$
$\boldsymbol{\eta}=\frac{\text { load }}{\text { effort }} \div \frac{\text { distance moved by effort }}{\text { distance moved by joad }} \times 100 \%=(M . A \div V . R) \times 100 \%$
$\boldsymbol{\eta}=\frac{\mathrm{M} . \mathrm{A}}{\mathrm{V} . . \mathrm{R}} \times 100 \%$

## Factors that affect efficiency of machines.

A machine that is $100 \%$ efficient is called an ideal machine.

Machines are never 100\% efficient because of the following factors:
(i) Friction in the movable parts of the machine.
(ii) Lifting useless loads which are part of the machine.

## Improving efficiency of a machine:

(i) By lubricating/oiling the movable parts of the machine to reduce friction.
(ii) By using light parts when making the machine eg alloys.

## Types of simple machines

These include levers, pulleys, inclined planes, wages, screws, wheels and axle, gears.

## LEVERS

A lever is a simple machine which has a turning point called fulcrum/pivot.
There are three classes of levers namely;
(a) $1^{\text {st }}$ class levers.
(b) $2^{\text {nd }}$ class levers.
(c) $3^{\text {rd }}$ class levers.

## Note: They may be recalled using the synonym PLE

## $1^{\text {st }}$ class levers

These are levers that have the pivot between the effort and the load.
Examples of $1^{\text {st }}$ class levers

- The children's see-saw.
- Pair of scissors.
- The crow-bar
- The beam-balance.
- Pair of pliers.

- Garden shears.
- Spade


Crow-bar


## $2^{\text {nd }}$ class lever

This is a lever that has the load between the pivot and the effort.

Examples of $2^{\text {nd }}$ class levers

- The wheel barrow.
- Nut crackers.
- Bottle opener.
- Staplers
- Nut cracker
- Nail clippers

(a) Wheel barrow

(c) Bottle opener.



## 3rd class lever

This is a lever where the effort is between the pivot and load.
Examples of 3 rd class levers

- Pair of tongs.
- Forceps.
- The fore arm.
- Shovels
- Fishing rods
- Human fore arms
- Legs
- Tweezers


## VELOCITY RATIO OF A LEVER:


velocity ratio of a lever $=\frac{\text { distance of effort from pivot }}{\text { distanc of load from pivot }}$

## EFFICIENCY OF A LEVER:

The efficiency of a lever is improved by increasing the distance of the effort from the pivot or turning point.

## Force multipliers and speed multipliers

Machines can make work easier by reducing the amount of force necessary to move an object or increasing the speed of an object relative to the force applied to it.
Force multipliers are devices that reduce the amount of force necessary to move an object. Force multipliers are useful for lifting heavy objects or doing other things that require large amounts of force. Some examples of force multipliers are inclined planes and most levers.

Speed multipliers are devices that increase the speed of, or distance travelled by, an object. Although more force than usual is required to move the object in these cases, the extra force is changed into more kinetic energy. Speed multipliers are useful when an object needs to move a further distance or at a higher speed. Some examples are wheels and axles and third class levers.

## Simple and complex machines

Machines can be either simple or complex. Simple machines are machines that only use one type of machine, such as a lever or an inclined plane.

Complex machines are machines that use multiple types of machines together. A door, for example, uses both a wheel and axle (the doorknob) and a lever (the door itself).

## PULLEYS

A pulley is a grooved wheel mounted on a block.
A string or rope passes around the pulley and it is held in place by the groove.


An effort, E is applied on one of the rope to produce tension, T in the rope such that:
effort $=$ tension $(E=T)$.
The tension in the rope is constant or uniform and acts in both directions along the rope. The load, L is supported by the total tension in the sections of the rope attached to the pulley on which the load is fixed.

Load, $\mathbf{L}=\mathbf{n T}$ where n is number of sections of the rope supporting the load.

## NOTE:

L can be less than $\mathrm{nT}(\mathrm{L}<\mathrm{nT})$ due to the weight of the pulleys.
$\mathrm{L}+\mathrm{W}=\mathrm{nT}$, where W is weight of the pulleys.
Note: But in most cases the pulleys will be taken to be weightless

## Types of pulley systems

## Single fixed pulley.

It consists of only one pulley which is fixed.


Tension, $\mathrm{T}=\mathrm{E}$
At equilibrium, $L=T$
Mechanical Advantage of a single fixed pulley.
M. $A=\frac{\text { Load }}{\text { effort }}=\frac{L}{E}$

But $\mathrm{L}=\mathrm{T}=\mathrm{E}$
M. $A=\frac{T}{T}=1$

Velocity ratio of a single fixed pulley.
V. $R=\frac{\text { Distance moved by effort }}{\text { Distance moved by load }}=\frac{\mathrm{x}}{\mathrm{x}}$
V. $\mathrm{R}=1$

## Advantage of using a single fixed pulley

The effort, E used to lift a load, L may be greater than the load, L itself due to friction between the groove and the rope (i.e. $\mathrm{E} \geq \mathrm{L}$ ), however, it is easier to pull downwards than to lift upwards when lifting loads because part of the operator's weight can form the effort $E$.
Therefore, the single fixed pulley eases work by changing the direction of application of the effort.

## 2. Single movable pulley

It consists of one moving pulley.


## At equilibrium

If the effort applied to the free end of the rope is E , then
The total upward force on the pulley is 2 E since two parts of rope support it.

$$
\text { Thus, } \mathrm{L}=2 \mathrm{E}
$$

$$
\text { M. A }=\frac{\mathrm{L}}{\mathrm{E}}=\frac{2 \mathrm{E}}{\mathrm{E}}=2
$$

Therefore, mechanical advantage of the single movable pulley is;
M. $A=\frac{L}{E}=\frac{2 T}{T}=2$

Velocity ratio of single movale pulley.
$\mathrm{V} . \mathrm{R}=\frac{\text { Distance moved by effort }}{\text { Distance moved by load }}=\frac{2 \mathrm{x}}{\mathrm{x}}=2$
Note: For every 1 m moved by the effort, each section of the rope supporting the load offers $\frac{1}{2}$ of a metre.
The system in figure (ii) is the same as the single movable pulley in figure (i) except that the effort is applied downwards. This makes it more convenient since part of the operator's weight can form the effort E
$V . R=2$

## Block and tackle single pulley system

It consists of two or more pulleys in two blocks mounted independently on the same axle. One block is fixed and the other is moving. The load is fixed on the lower movable block.
The upper fixed block has a number of pulleys as stated below in comparison with the lower movable block
When the total number of pulleys in the system is even, then -:
Number of pulleys in the upper block $=$ number of pulleys in the lower block.
When the total number of pulleys in the system is odd, then -:
Number of pulleys in the upper fixed block exceeds number of pulleys in lower movable block by 1 .
System of 2 pulleys


At Equilibrium ,
$\mathrm{L}=2 \mathrm{~T}, \mathrm{E}=\mathrm{T}$
$M . A=\frac{L}{E}=\frac{2 T}{T}=2$

## System of 3 pulleys



At equilibrium
$\mathrm{L}=3 \mathrm{~T}, \mathrm{E}=\mathrm{T}$
M. $A=\frac{L}{E}=\frac{3 T}{T}=3$

## System of 4 pulleys

At equilibrium
Load $\mathrm{L}=4 \mathrm{~T}$ and $\mathrm{E}=\mathrm{T}$

M. A $=\frac{L}{E}=\frac{4 T}{T}=4$

## System of 5 pulleys

At equilibrium
$\mathrm{L}=5 \mathrm{~T}$ and $\mathrm{E}=\mathrm{T}$
M. $A=\frac{L}{E}=\frac{5 T}{T}=5$


In the block and tackle system of pulleys:V. $\mathrm{R}=$ number of pullys in the system $=$ number of ropes/strings supporting the lower movable block/load.

## EXAMPLES:

1. For each of the pulley systems shown below, calculate
(i) velocity ratio
(ii) mechanical advantage
(iii) efficiency
2. Draw a diagram of a single string block and tackle system with a velocity ratio of 6 .

Calculate its efficiency if an effort of 1500 N is required to raise a load of 4990N. The weight of the lower block and the pulleys is 10 N .
3. A block and tackle pulley system has a velocity ratio of 4 . If its efficiency is $75 \%$. Find the
(a) mechanical advantage
(b) total load that can be lifted with an effort of 500 N
(c) work done if the load is lifted through a vertical distance of 4.0 m .
(d) the average rate of doing work if the work is done in 2 minutes.
4. A pulley system of V.R 3 supports a load of 20 N . Given that the tension in each string is 8 N , calculate
(i) the effort required to raise a load,
(ii) the mechanical advantage,
(iii) the efficiency of the pulley system,
(iv) the distance moved by the effort if the load moves through a distance of 2 m ,
(v) the weight of lower pulley.

SOLUTION
(i) $\mathrm{E}=\mathrm{T}=8 \mathrm{~N}$
(ii) $\mathrm{M} . \mathrm{A}=\frac{\mathrm{L}}{\mathrm{E}}=\frac{20}{8}=2.5$
(iii) Efficiency, $\tau=\frac{\text { M.A }}{V . R} \times 100=\frac{2.5}{3.0} \times 100=831 / 3 \%$
(iv) V.R $=\frac{\mathrm{dE}}{\mathrm{dL}}$

$$
3=\frac{\mathrm{de}}{2}
$$

$$
\therefore \mathrm{dE}=3 \times 2=6
$$

(v) $3 \mathrm{~T}=\mathrm{L}+\mathrm{W}$
$3 \times 8=20+W$
Therefore weight of the pulley $\mathrm{W}=24-20=4 \mathrm{~N}$
5. A single stringed pulley system is as shown below.


A load of 20 N is raised by an effort of 8 N . If the system is friction less, find the mass of the lower pulley.

## SOLUTION:

Upward forces = down ward forces
$3 \mathrm{~T}=20+\mathrm{W}$
$3 \times 8=20+W$
$W=(24-20)$
$\mathrm{W}=4 \mathrm{~N}$
Mass of the lower pulley, $\mathrm{m}=\frac{\mathrm{w}}{\mathrm{g}}=\frac{4}{10}=0.4 \mathrm{~kg}$.
6. An effort of 50 N is required to raise a load of 200 N using a pulley system of velocity ratio 5 .
(i) Draw a diagram to show the pulley system.
(ii) Find the efficiency of the system.
(iii) Calculate the work wasted when the load is raised through 120 cm .
(iv) Give 2 reasons why efficiency of your pulley system is always less than $100 \%$. SOLUTION:
See diagram.

$$
\begin{aligned}
& \qquad \mathrm{M} . \mathrm{A}=\frac{\mathrm{L}}{\mathrm{E}}=\frac{200}{50}=4, \mathrm{~V} \cdot \mathrm{R}=5 \\
& \boldsymbol{\eta}=\frac{\mathrm{M} \cdot \mathrm{~A}}{\mathrm{~V} \cdot \mathrm{R}} \times 100=\frac{4}{5} \times 100=80 \% \\
& \qquad \mathrm{dL}=120 \mathrm{~cm}=1.2 \mathrm{~m}
\end{aligned} \begin{aligned}
& \mathrm{V} . \mathrm{R}=\frac{\mathrm{dE}}{\mathrm{dL}}=\frac{\mathrm{dE}}{1.2}=5 \therefore \mathrm{dE}=1.2 \times 5=6 \mathrm{~m} \\
& \text { Work in put }=\mathrm{E} \times \mathrm{dE}=50 \times 6=300 \mathrm{~J} \\
& \text { Work output }=\mathrm{L} \times \mathrm{dL}=200 \times 1.2=240 \mathrm{~J} \\
& \text { Work wasted }=\text { work in put }- \text { work output } \\
& \quad=300-240 \\
& \quad=60 \mathrm{~J}
\end{aligned}
$$

Exercise

1. 2013 p2 no. 1 (a) - (c)
2. Longhorn exercise 4.1 page 98-100

## Note:

Total work done $=$ useful work done + useless work done (work input) $=$ (work output) + (work wasted)

Work wasted $=$ work input - work output.

## APPLICATIONS/USES OF PULLEYS:

- Pulleys are used in cranes to raise materials to higher levels at construction sites.
- They are used in lifts to transport passengers upstairs or downstairs.
- They are used in elevators to move cargo from one point to another.
- They are used in window curtain rails to draw curtains.
- They are used in raising and lowering of a flag.
- Raising water from a well.


## Experiment: To investigate the V.R of a Block and Tackle System.

A block and tackle system with two pulleys in the lower block and two pulleys in the upper block as shown below.


A block-and-tackle system with a mass hanging at the lower block arranged.
The number of sections of the string supporting the lower block is counted.
The load is raised by any given length, $l$, by pulling the effort downwards.
The distance, $e$, moved by the effort is measured.
The experiment is repeated by increasing the distance moved by the effort.
The values are recorded in the table as shown below.

| $e(\mathrm{~cm})$ | $l(\mathrm{~cm})$ |
| :--- | :--- |
| 10 |  |
| 15 |  |
| 20 |  |
| 25 |  |

A graph of $e$ against $l$ is plotted
The gradient $\frac{\Delta e}{\Delta l}$ which is the velocity is found to be 4 which is the same as the number of sections of the string supporting the lower block

## Precaution

The weight of the block in the lower section of the system has to be considered as this increases the load to be lifted.
Experiment: To investigate the variation of M.A /efficiency of a Pulley System with the load.


The pulley system is arranged with the string appropriately assembled.
A load is fixed to the lower block.
A spring balance is fixed at the free end of the string.
By pulling on the spring balance, the minimum effort E , required to lift the load, is found and noted. The procedure is repeated for several other values of the load and a table as shown below is filled.

| Load(N) | Effort, E(N) | M. $\mathrm{A}=\frac{\mathrm{L}}{\mathrm{E}}$ |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

A graph of mechanical advantage against the load is plotted.
Mechanical advantage

## Conclusion

As the lord increases also the mechanical advantage increases.

## Note

When the load is less than the weight of the lower pulley block, most of the effort is used to overcome the frictional forces at the axle and the weight of the lower pulley block. That is, the effort does useless work. However, when the load is larger than the weight of the lower block, the effort is used to lift the load. This shows that the effort is more efficient when lifting a load that is greater than the weight of the lower block.

Note: The graph of efficiency against load is the same as that of the M.A against the load but steeper.

## INCLINED PLANE



This is wooden plank inclined at an angle to the ground.
Examples of inclined planes;

- Stair case
- Sloping and winding roads up mountainous regions.




## Working of the inclined plane.

The effort, E raises the load (box), L through a vertical height BC by pulling it along the inclined plane from point $A$ at the ground to point $B$ at the required height.
AB - length of inclined plane, l.
L-Load / weight
E - Effort (pull / push)
$\theta$ - Angle of inclination of the plane to the ground
$B C$-vertical height, h.
Velocity of the inclined plane.
V. $\mathrm{R}=\frac{\text { distance covered by effort, } \mathrm{E}}{\text { distance covered by the load, } \mathrm{L}}$
$\mathrm{V} . \mathrm{R}=\frac{\text { length of the inclined plane, } l}{\text { vertical height of the plat form, } \mathrm{h}}$
Therefore

$$
\text { V. } \mathrm{R}=\frac{l}{\mathrm{~h}}
$$

Examples

1. An effort of 50 N is used to haul a 300 N box along an incline which rises vertically 1 m for every 8 m distance along the plane. Find
(i) the velocity ratio
(ii) the mechanical advantage
(iii) the efficiency
(iv) State the factors in this arrangement that make the efficiency become less than $100 \%$

Solution
(i) The velocity ratio $=\frac{8}{1}=8$
(ii) Mechanicaladvantage $=\frac{\text { Load }}{\text { Effort }}=\frac{300}{50}=6$

$$
\text { (iii) Efficiency }=\frac{\text { Mechanicaladvantage }}{\text { velocityratio }} \times 100=\frac{6}{8} \times 100=75 \%
$$

2. Below is an inclined plane used to lift a load from $R$ to $P$ as shown.


Determine
(i) mechanical advantage
(ii) velocity ratio
(iii) Efficiency of the machine.
(i) $\mathrm{M} . \mathrm{A}=\frac{\text { Load }}{\text { Effort }}=\frac{100}{90}=1.11$
(iii) Length of plane:

$$
l^{2}=5^{2}+10^{2}=125
$$

$$
\therefore \quad l=\sqrt{125}=11.18 \mathrm{~cm}
$$

$$
\mathrm{V} . \mathrm{R}=\frac{\text { distance moved by load }}{\text { vertical height of platformm }}=\frac{\mathrm{l}}{\mathrm{H}}=\frac{11.18}{10}=1.11
$$

(iii) $\quad \eta=\frac{M . A}{V . R} \times 100 \%=\frac{1.11}{1.117} \times 100 \%=99.37 \%$.

## ALTERNATIVE:

work input $=$ effort $\times$ distance moved by effort $=90 \times \frac{11.18}{100}=10.062 \mathrm{~J}$
work output $=$ load $\times$ distance moved by load $=100 \times \frac{10}{100}=10 \mathrm{~J}$

$$
\eta=\frac{\text { work output }}{\text { work input }} \times 100 \%=\frac{10}{10.062} \times 100 \%=99.38 \%
$$

3. (a) What is meant by first class lever?
(b) By means of a lever, an effort of 50 N moves a load of 200 N through a distance of 3 m .

If the effort moves a distance of 16 m ; calculate
(i) The mechanical advantage
(ii) The efficiency.

SOLUTION
(a) A first class lever is one where the pivot is in between the load and effort.
(b)
(i) $\mathrm{M} . \mathrm{A}=\frac{\mathrm{L}}{\mathrm{E}}=\frac{200}{50}=4$
(ii) $\quad \eta=\frac{M \cdot A}{V \cdot R} \times 100 \%=\frac{4}{V \cdot R} \times 100 \%$

$$
\text { but V. } \mathrm{R}=\frac{\mathrm{D}_{\mathrm{E}}}{\mathrm{D}_{\mathrm{L}}}=\frac{16}{3}=5.333
$$

$$
\therefore \eta=\frac{4}{5.333} \times 100 \%=75 \%
$$

## SCREWS

Pitch is the distance between two successive threads measured along the axis of the screw.
It is equal to the distance moved by the load when the screw is rotated through one complete turn.
Examples of a screw jack


When the screw is turned through one complete revolution, it advances a distance equal to one pitch
P - Pitch
$r$-radius of rotation (effort distance)
V. R of screw $=\frac{\text { circumference of one rotation }}{\text { pitch }}=\frac{2 \pi r}{\mathrm{p}}$
$\therefore \quad$ V.R $=\frac{\text { Circumference of circle made by } \mathrm{E}}{\text { Pitch }}$
Usually, the effort is applied on a tommy bar some distance r from the axis of the screw. So

$$
\mathrm{V} . \mathrm{R}=\frac{2 \pi \mathrm{r}}{\mathrm{pitch}}
$$

Example:
A screw jack is found to be $70 \%$ efficient. If an effort of 20 N is used to lift a vehicle of 5000 N and the pitch of the screw is 2 mm , what is the length of the tommy bar?

Solution:
Let $\mathrm{r}=$ length of the tommy bar
Then velocity ratio, V. $\mathrm{R}=\frac{2 \pi \mathrm{r}}{\text { pitch }}=\frac{2 \pi \mathrm{r}}{2}=\pi \mathrm{r}$
Mechanical advantage, M.A $=5000 / 20=250$
Now, efficiency = M.A/V.R
$\therefore \quad 70 / 100=250 / \pi r$
$\therefore \quad \mathrm{r}=\frac{250 \times 100}{\pi \times 70}=113.6 \mathrm{~mm}$
Example
The pitch of a bolt is 1 mm . To tighten the bolt, a worker uses a spanner of a long arm of effective length 80 cm . Calculate the velocity ratio of the Spanner. (5028)

## Example

A screw has a pitch of 5 mm . If an effort of 30 N is rotated through one turn of radius 50 cm to lift a load of 750 N , find,
(i) the M.A,
(ii) the V.R,
(iii) the efficiency.

Solution:
(i) M. A $=\frac{\text { Load }}{\text { Effort }}=\frac{750 \mathrm{~N}}{30 \mathrm{~N}}=25$ radius, $\mathrm{r}=50 \mathrm{~cm}=500 \mathrm{~mm}, \quad$ pitch, $\mathrm{P}=5 \mathrm{~mm}$
(ii) $\mathrm{V} . \mathrm{R}=\frac{2 \pi \mathrm{r}}{\mathrm{P}}=\frac{2 \times 3.14 \times 500}{5}=628$
(iii) $\eta=\frac{M . A}{V . R} \times 100 \%=\frac{25}{628} \times 100 \%=3.98 \%$.

Efficiency of a screw.
Screws have low efficiency because of the great fiction offered by their threads (i.e. screws are rough)
The large friction in screws is necessary to enable them hold firmly what they are supporting. Questions

1. A machine of velocity ratio 5 is used to raise a load of weight is 200 N . If an effort of 50 N is applied, calculate
(i) its efficiency.
(ii) the useless work.
2. A trolley of weight 10 N is pulled from the bottom to the top of the inclined plane by a steady force of 2 N . If the height and distance moved by the force are 2 m and 20 m respectively, calculate;
(i) M.A
(ii) V.R
(iii) Efficiency

## WHEEL AND AXLE

It consists of a large diameter wheel attached to an axle of smaller diameter.


The Effort is applied to one end of a rope passing over the wheel of radius, R while the load is at the end of another rope passing over the axle of radius, $r$.
After one complete turn, the Effort (E) moves through a distance equal to $2 \pi R$ and the load is raised through a distance $2 \pi r$.
V. R of the wheel and axle $=\frac{\text { Distance moved by effort }}{\text { distance moved by load }}$
V. $R=\frac{2 \pi R}{2 \pi r}=\frac{R}{r}=\frac{\text { radius of the wheel }}{\text { radius of the axle }}$

## Example

The system below is a wheel and axle of radii 40 cm and 4 cm respectively. Assuming that the efficiency of the above system is $45 \%$.


Find
(i) The effort required to raise the load.
V. $\mathrm{R}=\frac{\mathrm{R}}{\mathrm{r}}=\frac{40 \mathrm{~cm}}{4 \mathrm{~cm}}=10$

Efficiency $=\frac{\text { M.A }}{\text { V.R }} \times 100$

$$
45=\frac{\mathrm{M} \cdot \mathrm{~A}}{10} \times 100
$$

M. A $=\frac{45 \times 10}{100}=4.5$
M. $A=\frac{\text { Load }}{\text { effort }}=\frac{1000}{\mathrm{E}}=4.5$

Effort, $\mathrm{E}=\frac{1000}{4.5}=222.2 \mathrm{~N}$.
(ii) The energy wasted when the effort moves through 1760 cm .

Work input $=\mathrm{E} \times \mathrm{dE}=\mathrm{E} \times 2 \pi \mathrm{R}=222.2 \times 2 \pi \times \frac{40}{100}=177.76 \pi \mathrm{~J}$
Work output $=\mathrm{L} \times \mathrm{dE}=\mathrm{L} \times 2 \pi \mathrm{r}=1000 \times 2 \pi \times \frac{4}{100}=80 \pi \mathrm{~J}$

$$
\text { Work wasted }=(177.76-80) \pi=97.76 \pi \mathrm{~J}=306.9664 \mathrm{~J}
$$

## Combination of simple machines


$\mathrm{V} . \mathrm{R}=$ Product of the individual velocity ratios $=\frac{a}{b} \times \frac{c}{d}$

Example
A bicycle has a driving gear wheel of radius 10 cm with 24 teeth. The driven gear wheel has a radius of 4 cm with 8 teeth. If a force of 300 N is used to drive a load of 75 N , determine:
(a) (i) the velocity ratio for the gear wheel system. (1/3)
(ii) the mechanical advantage. (0.25)
(iii) efficiency.(75\%)
(b) What is the effect of increasing the number of teeth on the driving wheel. The velocity ratio will be decreased and the arrangement will speed up the rotation.
(c) Why is it unrealistic to have both wheels having the same radius and equal number of teeth. There will be no increase on mechanical advantage. This means that the effort applied must be very large even on a flat ground. The friction will cause the load to be less than effort.

## Example:

An effort E is used to just lift a load of weight 420 N using the arrangement shown below


If the efficiency of the arrangement is $70 \%$, find
(i) the velocity ratio of the system
(ii) the effort E.

## Solution:

Overall velocity ratio $=$ velocity ratio of the pulley system x velocity ratio of the lever

$$
\begin{aligned}
& \text { Overall velocity ratio }=2 \times \frac{5}{3}=\frac{10}{3}=3.33 \\
& \text { Efficiency }=\frac{M . A}{V . R} \times 100=\frac{420 \times 3}{E \times 10} \times 100 \% \\
& \therefore 70=\frac{420 \times 3}{E \times 10} \times 100 \\
& \therefore E=\frac{420 \times 3}{70 \times 10} \times 100=180 \mathrm{~N}
\end{aligned}
$$

## GEARS

In gears, the effort is applied to one wheel which is called the driving wheel. The other wheel to which the load is connected is called the driven wheel.


Velocity ratio $=\frac{\text { speed of rotation of driving wheel }}{\text { speed of rotation of driven wheel. }}$
$\mathrm{V} . \mathrm{R}=\frac{\text { Number of teeth in a driven wheel }}{\text { number of teeth in driving wheel. }}$
If gear A turns its teeth, it interlocks with those of $B$ and make it turn in the opposite direction.
N.B: The fastest turning gear is that with the smallest number of teeth.

Examples
A wheel and axle machine has 120 teeth in the driven gears and 40 teeth in the driven gear.
Calculate
(i) It's V.R.

$$
\begin{gathered}
\text { V.R }=\frac{\text { number ot teeth in driven gear }}{\text { over number of teeth in driving gear. }} \\
\quad=\frac{120}{40}=3
\end{gathered}
$$

(ii) Its M.A(if the machine is $80 \%$ efficient)

Efficiency $\eta=\frac{M \cdot A}{V . R} X 100$

$$
\begin{gathered}
80=\frac{\text { M.A }}{3} \mathrm{X} 100 \\
\text { M.A }=\frac{80 \mathrm{X} 3}{100}=2.4
\end{gathered}
$$

## HYDRAULIC PRESS

It works on the principle that pressure transmitted through an incompressible liquid/ fluid is the same everywhere in the fluid.


Pressure $p=\frac{\operatorname{force}(F)}{\operatorname{area}(A)}$
F = P X A
Therefore E $=\mathrm{PX} \mathrm{A} \mathrm{A}_{1}=\mathrm{P} \mathrm{X} \pi r^{2}$
And load $L=P X A_{2}=P X \pi R^{2}$
M.A $=\frac{\text { load }}{\text { effort }}=\frac{\mathrm{P} \pi \mathrm{R}^{2}}{\mathrm{P} \pi \mathrm{r}^{2}}$
M.A $=\frac{\mathrm{R}^{2}}{\mathrm{r}^{2}}$

When E and L move through a distance $x$ and $y$ respectively, the volumes are pressed by small piston is equal to volume raised up in the; large piston.
$A_{1} x=A_{2} y=\pi r^{2} x=\pi R^{2}=\frac{x}{y}=\frac{\pi R^{2}}{\pi r^{2}}$
$V \cdot R=\frac{\mathrm{R}^{2}}{\mathrm{r}^{2}}$

## Example

A hydraulic hoist has a main cylinder diameter of 30 cm and a pump cylinder diameter of 1 cm .
Calculate
(i) V.R
(ii) The maximum load it can raise
$\qquad$
(iii) M.A (given that the force applied on the piston pump 70N and efficiency equal 80\%)
(iv) The efficiency of the hydraulic press is $60 \%$. Find the load raised if an effort of 200 N applied on a piston of radius 5 cm and the load is pressed on the piston of radius 30 cm .

END.

